Limits and Continuity

1. For the following function:

   Find:

   $$(i)\ \lim_{x\to 4^-} f(x)$$
   $$(ii)\ \lim_{x\to 4^+} f(x)$$
   $$(iii)\text{Does}\lim_{x\to 4}\text{exist? Why or why not?}$$
   $$(iv)\text{Is}f(x)\text{continuous at 4? Why or why not?}$$

2. For:

   $$f(x)=\begin{cases} x^3+4 & \text{if } x<2;\\ \frac{x^3-8}{x-2} & \text{if } x>2;\\ 8 & \text{if } x=2. \end{cases}$$

   find

   $$(i)\ \lim_{x\to 2^-} f(x)$$
   $$(ii)\ \lim_{x\to 2^+} f(x)$$
   $$(iii)\ \text{Is}f(x)\text{continuous at 2? Why or why not?}$$

3. For what value of c is

   $$f(x)=\begin{cases} \frac{x^2-4x+5}{x+1} & \text{if } x<-1;\\ -cx^2+1 & \text{if } x\geq 1. \end{cases}$$

   continuous at -1?